

Many-to-Many Disjoint Path Covers in DCell

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Abstract

Data center networks (DCNs for short) become more and more important with the development of cloud computing. For any integers $k \geq 0$ and $n \geq 2$, the k -dimensional DCell with n -port switches, $D_{k,n}$, has been proposed for one of the most important DCNs as a server centric DCN structure. In this paper, we give a construction scheme for many-to-many r -disjoint path covers in $D_{k,n}$ with $1 \leq r \leq \lfloor (n+k-1)/2 \rfloor$.

Keywords

DCell; Disjoint Path Cover; Hamiltonian Path

Introduction

The performance improvement of cloud computing faces great challenges of how to connect a large number of servers in building a data center network with promising performance [1]. Guo et al. have recently proposed a novel data center network called DCell [2], which addresses the needs of a data center. Its desirable properties include doubly exponential scaling, high network capacity, large bisection width, small diameter and high fault-tolerance.

The many-to-many node-disjoint paths covering (DPC for short) problem in DCNs have been an important subject because DPC routing would offer many benefits such as fault tolerance, software testing, etc [3,4,5].

A DCN can be represented by a simple graph $G = (V(G), E(G))$, where $V(G)$ represents the node set and $E(G)$ represents the edge set, and each node represents a server and each edge represents a link between servers (switches can be regarded as transparent network devices [2]). For a set $S = \{s_1, s_2, \dots, s_r\}$ of r source nodes (sources for short) and a set $T = \{t_1, t_2, \dots, t_r\}$ of r sink nodes (sinks for short) in $V(G)$, the many-to-many r -DPC problem is to determine whether there exist r -node disjoint paths, each path joining a source and a sink, these r paths cover $V(G)$. There are paired and unpaired types of many-to-many r -DPC problems. In the paired type, each source should be joined to a specific sink, that is, s_j should be joined to t_j for $1 \leq j \leq r$.

In this paper, we discuss about some interesting properties of paired many-to-many DPC in DCell networks. Then, we give a construction scheme for paired many-to-many r -DPCs in DCell networks with $1 \leq r \leq \lfloor (n+k-1)/2 \rfloor$.

This work is organized as follows. Section 2 provides the preliminary knowledge. Some basic properties are given in Section 3. We make a conclusion in Section 4.

Preliminaries

In this section, we provide some graph theoretical terminology and give the definition of DCell.

Let G_1 and G_2 be two subgraphs of G . We use $G_1 \cup G_2$ to denote the subgraph induced by $V(G_1) \cup V(G_2)$ of G . For $U \subseteq V(G)$, we use $G[U] = (U, E')$ to denote the subgraph induced by U in G where $E' = \{(u, v) \in E(G) \mid u, v \in U\}$. A path in a graph is a sequence of nodes, $P = \langle u_0, u_1, \dots, u_j, \dots, u_{n-1}, u_n \rangle$, in which no nodes are repeated and u_j, u_{j+1} are adjacent for any integer $0 \leq j < n$. The reverse of P is $\langle u_n, u_{n-1}, \dots, u_1, u_0 \rangle$, denoted by P^{-1} . We use $V(P)$ to denote the set of all nodes appearing in P and $E(P)$ to denote the set of all edges appearing in P . The

path, starting from u_i and ending with u_j in a path Q , can be denoted by $P(Q, u_i, u_j)$. The length of a path P , $l(P)$, is the number of edges in P . We also write the path $\langle u_1, u_2, \dots, u_k \rangle$ as $\langle u_1, Q_1, u_i, u_{i+1}, \dots, u_k \rangle$ where Q_1 is the path $\langle u_2, u_3, \dots, u_{i-1} \rangle$. Hence, it is possible to write a path as $\langle u_1, Q, u_2, u_3, \dots, u_k \rangle$ if $l(Q) = 0$.

A path in a graph G containing every node of G is called a Hamiltonian path (HP). $HP(u, v, G)$ can denote a Hamiltonian path beginning with a node u and ending with another node v in graph G . A graph G is called paired (resp. unpaired) many-to-many r -disjoint path coverable (DPC-able for short) if $2r \leq |V(G)|$, G has a paired (resp. unpaired) r -DPC for any set S of r sources and any set T of r sinks in G such that $S \cap T = \emptyset$. A paired many-to-many r -DPC-able graph is unpaired many-to-many r -DPC-able.

DCell uses a recursively defined structure to interconnect servers. Each server connects to different levels of DCell through multiple links. For any integers $k \geq 0$ and $n \geq 2$, we use $D_{k,n}$ to denote a k -dimensional DCell with n -port switches. $D_{0,n}$ is a complete graph on n nodes for any integer $n \geq 2$. We use N_0 to denote the number of nodes in $D_{0,n}$ with $N_0 = n$. Furthermore, we use N_k to denote the number of nodes in $D_{k,n}$ for $k \geq 1$, where $N_k = N_{k-1}(N_{k-1} + 1)$. The node u of $D_{k,n}$ can be labeled by a tuple $[u_k, \dots, u_i, \dots, u_0]$ such that $u_0 \in \{0, 1, \dots, n-1\}$ and $u_i \in \{0, 1, \dots, N_{i-1}\}$ for all $i = 1, 2, \dots, k$.

The $D_{k,n}$ is defined recursively as follows. (1) $D_{0,n}$ is a complete graph consisting of n nodes. (2) For any integer $k \geq 0$, $D_{k,n}$ is built from $N_{k-1} + 1$ disjoint copies $D_{k-1,n}$, according to the following steps. (2.1) We use $D_{k-1,n}^i$ to denote the graph obtained by prefixing the label of each node of one copy of $D_{k-1,n}$ with i for $i = 0, 1, \dots, N_{k-1}$. (2.2) Node $u = [u_k, u_{k-1}, \dots, u_0]$ in $D_{k-1,n}^{u_k}$ is adjacent to node $v = [v_k, v_{k-1}, \dots, v_0]$ in $D_{k-1,n}^{v_k}$ if and only if $u_k = v_0 + \sum_{j=1}^{k-1} (v_j N_{j-1})$ and $v_k = u_0 + \sum_{j=1}^{k-1} (u_j N_{j-1}) + 1$.

By definition of DCell, we have the following Lemma.

Lemma 1. There exists a unique external edge, denoted by $e(D_{k-1,n}^\alpha, D_{k-1,n}^\beta)$, which connects $D_{k-1,n}^\alpha$ to $D_{k-1,n}^\beta$ with $\alpha \neq \beta$.

For any integer $d \geq 1$, when two adjacent nodes u and v have a leftmost differing element at the position d , we say that v is the d -neighbor of u or the edge (u, v) is an edge of dimension d . We use $(u)^d$ to denote the d -neighbor of u if $d \geq 1$. For all integer $0 \leq i \leq n-1$, let $z_i = [u_k, \dots, u_1, i]$, we say that z_i is a 0-neighbor of u or the edge (u, z_i) is an edge of dimension 0 if $z_i \neq u$. Let $(u)^0 = \{z_i \mid 0 \leq i \leq n-1 \text{ and } z_i \neq u\}$, we use $(u)^0$ to denote the 0-neighbors of u . For any node $u = [u_k, u_{k-1}, \dots, u_0]$ in $D_{k,n}$, we use $(u)_i$ to denote the i -element u_i of u .

Main Results

Some properties of the DCell networks have received considerable attentions in the literature [2,7].

Theorem 2. For any two integers $k \geq 0$ and $n \geq 2$, the number of servers in $D_{k,n}$ satisfies $N_k \geq (n+1/2)^{2^k} - 1/2$.

Theorem 3. For any two integers $k \geq 0$ and $n \geq 2$, $D_{k,n}$ is Hamiltonian-connected except for $D_{1,2}$.

By Theorem 3, we have the following three Lemmas.

Lemma 4. $D_{0,n}$ is a paired many-to-many r -disjoint path coverable with $1 \leq r \leq \lfloor (n-1)/2 \rfloor$.

Lemma 5. For any two integers $k \geq 0$ and $n \geq 2$, $D_{k,n}$ is paired many-to-many 1-disjoint path coverable, except for $D_{1,2}$.

Lemma 6. For any two integers $k > 2$ and $n = 2$, or $k > 0$ and $n > 2$. Let $G = D_{k,n}[V(\bigcup_{\theta=1}^m D_{k-1,n}^{\omega_\theta})]$ be a sub graph of $D_{k,n}$ with $2 \leq m \leq N_{k-1}$. For any two distinct nodes u and v in G with $(u)^k, (v)^k \notin V(G)$, $(u)_k \neq (v)_k$, $(u)_k = \omega_1$, and $(v)_k = \omega_m$, there exists a Hamiltonian path of G joining u and v .

Lemma 7. $D_{1,4}$ is a paired many-to-many r -disjoint path coverable with $1 \leq r \leq 2$.

Proof. The Lemma is verified by a computer program.

Lemma 8. For any integer $n \geq 4$, $D_{1,n}$ is a paired many-to-many r -disjoint path coverable with $1 \leq r \leq \lfloor n/2 \rfloor$.

Proof. By Lemma 7, the lemma holds for $D_{1,4}$. For any integer $n > 4$, the lemma is proved by induction on r and we omitted for limitation of length.

Lemma 9. For any integers $k \geq 0$ and $n \geq 2$, $D_{k,n}$ is paired many-to-many r -disjoint path coverable with $1 \leq r \leq \lfloor (n+k-1)/2 \rfloor$ and $n+k$ is even, except for $D_{1,2}$.

Proof. By Lemma 4, the lemma holds with $n=0$. By Lemma 5, the lemma holds for $r=1$ with $k \geq 0$ and $n \geq 2$, except for $D_{1,2}$. We will prove this lemma by induction on the dimension k of $D_{k,n}$ for other cases. We assume that $D_{k,n}$ is paired many-to-many r -DPC-able for $\tau \geq 1$ and $n \geq 3$ or $\tau \geq 2$ and $n=2$ with $1 \leq r \leq \lfloor (n+\tau-1)/2 \rfloor$ and $n+\tau$ is even.

In the following, we will prove the $D_{\tau+1,n}$ is paired many-to-many r -DPC-able. That is, given a set of $r+1$ sources $S = \{s_1, s_2, \dots, s_{r+1}\}$ and a set of $r+1$ sinks $T = \{t_1, t_2, \dots, t_{r+1}\}$ in $D_{\tau+1,n}$, we will prove that there exist $r+1$ disjoint paths joining S and T in $D_{\tau+1,n}$ with $1 \leq r \leq (n+\tau)/2$ and $S \cap T = \emptyset$, such that the union of these r paths covers $V(D_{\tau+1,n})$. For any integer $1 \leq i \leq r+1$, let $G_{s_i} = D_{\tau,n}^{(s_i)_1}$ and $G_{t_i} = D_{\tau,n}^{(t_i)_1}$. What's more, let $W_0 = \bigcup_{\theta=1}^{r+1} G_{s_i}$, $W_1 = \bigcup_{\theta=1}^{r+1} G_{t_i}$, $W_2 = W_S \cup W_T$, $W_3 = S \cup T$, $G_0 = D_{\tau+1,n}[V(W_2)]$, and $G_1 = D_{\tau+1,n}[D_{\tau+1,n}/G_0]$. Furthermore, for any integer $0 \leq j \leq N_\tau$, let $S_j = S \cap V(D_{\tau,n}^j)$, $T_j = T \cap V(D_{\tau,n}^j)$, and $R_j = S_j \cup T_j$. Then, let $p = \max\{|R_0|, |R_1|, \dots, |R_{N_\tau}|\}$. We can claim the following three cases with respect to m .

Case 1. $1 \leq p \leq r$. By Theorem 2, we have $N_\tau + 1 \geq 1/2 + (n+1/2)^{2^r} \geq 2(n+\tau) \geq 2(r+1)$. For any integer $1 \leq i \leq r+1$, find $u_i \in V(G_{s_i})$, $v_i \in V(G_{t_i})$, and $x_i, y_i \in V(G_1)$ such that $u_i, v_i \notin \{s_i, t_i\}$, $x_i = (u_i)^{\tau+1}$, $y_i = (v_i)^{\tau+1}$, and $(x_1)^1 \neq (x_2)^1 \neq \dots \neq (x_{r+1})^1 \neq (y_1)^1 \neq (y_2)^1 \neq \dots \neq (y_{r+1})^1$. What's more, for any integer $1 \leq i \leq r+1$, by Lemma 6, there exists a

Hamiltonian path Q_i joining $(u_i)^{\tau+1}$ and $(v_i)^{\tau+1}$ in $G = \begin{cases} D_{\tau,n}^{(x_i)_1} \cup D_{\tau,n}^{(y_i)_1} & \text{if } i \leq r, \\ G_0 \sqcup \bigcup_{i=1}^r (D_{\tau,n}^{(x_i)_1} \cup D_{\tau,n}^{(y_i)_1}) & \text{if } i = r+1. \end{cases}$

Case 1.1. $p=1$. We can confirm that $P_1 = \langle HP(s_1, u_1, G_{s_1})$

, $Q_1, HP(v_1, t_1, G_{t_1}) \rangle$, $P_2 = \langle HP(s_2, u_2, G_{s_2}), Q_2, HP(v_2, t_2, G_{t_2}) \rangle$, \dots , and $P_{r+1} = \langle HP(s_{r+1}, s_{\tau+1}, G_{s_{r+1}}) + (s_{i'}, s_{i'}'), Q_{\tau+1}, HP(t_{\tau+1}, t_{\tau+1}, G_{t_{\tau+1}}) \rangle$ are $r+1$ disjoint paths such that $V(\bigcup_{i=1}^{r+1} P_i) = V(D_{\tau+1,n})$.

Case 1.2. $2 \leq p \leq r$. For any integer $0 \leq i \leq N_\tau$ with $|R_i| > 0$, we use $\{s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_{|S_i|}}, t_{\beta_1}, t_{\beta_2}, \dots, t_{\beta_{|T_i|}}\}$ to denote R_i with $\alpha_1, \dots, \alpha_{|S_i|}, \beta_1, \dots, \beta_{|T_i|} \in \{1, 2, \dots, \tau+1\}$ and $\alpha_1 \neq \dots \neq \alpha_{|S_i|} \neq \beta_1 \neq \dots \neq \beta_{|T_i|}$. According to the induction hypothesis, these exist $|R_i|$ disjoint paths $P_{\alpha_1}, P_{\alpha_2}, \dots, P_{\alpha_{|S_i|}}, Q_{\beta_1}, Q_{\beta_2}, \dots, Q_{\beta_{|T_i|}}$ joining $\{s_{\alpha_1}, \dots, s_{\alpha_{|S_i|}}, v_{\beta_1}, \dots, v_{\beta_{|T_i|}}\}$ and $\{u_{\alpha_1}, \dots, u_{\alpha_{|S_i|}}, t_{\beta_1}, \dots, t_{\beta_{|T_i|}}\}$ such that the union of these $|R_i|$ paths covers $V(D_{\tau+1,n})$ for any integer $0 \leq i \leq N_\tau$. We can confirm that $P_1 = \langle P_1', Q_1, Q_1' \rangle$, $P_2 = \langle P_2', Q_2, Q_2' \rangle$, \dots , and $P_{r+1} = \langle P_{r+1}', Q_{\tau+1}, Q_{\tau+1}' \rangle$ are $r+1$ disjoint paths such that $V(\bigcup_{i=1}^{r+1} P_i) = V(D_{\tau+1,n})$.

Case 2. $r+1 \leq p < 2r$. The case is similar to the Case 1, so we skip it.

Case 3. $p \geq 2r$. Choose R_λ such that $|R_\lambda| = p$ with $0 \leq \lambda \leq N_\tau$. Let $q = 2r + 2 - p$. Then, we can claim the following three cases with respect to q .

Case 3.1. $q=0$. According to the induction hypothesis, there exist r disjoint paths P_1, P_2, \dots, P_r joining $\{s_1, s_2, \dots, s_r\}$

and $\{t_1, t_2, \dots, t_r\}$ in $D_{\tau, n}^\lambda$ such that $V(\bigcup_{i=1}^{r+1} P_i) = V(D_{\tau+1, n})$. Choose P_ϕ and P_φ such that and $t_{r+1} \in V(P_\varphi)$ with $\phi, \varphi \in \{1, 2, \dots, r\}$. Then, we can identify the following two cases with respect to ϕ and φ .

Case 3.1.1. $\phi = \varphi$. Let $Q_0 = P(P_\phi, s_{r+1}, t_{r+1})$. It is easy to verify that $l(P_\phi) \geq l(Q_0) + 2$. Choose four distinct nodes s_{r+1}^0 , s_{r+1}^1 , t_{r+1}^0 , and t_{r+1}^1 such that (s_{r+1}^0, s_{r+1}^1) , (s_{r+1}^1, s_{r+1}^0) , (t_{r+1}^0, t_{r+1}^1) , $(t_{r+1}^1, t_{r+1}^0) \in E(P_\phi)$. What's more, let $x = s_{r+1}$, $Q_1 = P(P_\phi, t_{r+1}, s_{r+1}^1)$, $Q_2 = P(P_\phi, t_{r+1}, s_{r+1}^0)$, and

$$y = \begin{cases} t_{r+1} & \text{if } t_{r+1} \in \{s_{r+1}^0, s_{r+1}^1\}, \\ s_{r+1}^1 & \text{if } l(Q_0) = l(Q_1) + 1 \geq 2, \\ s_{r+1}^0 & \text{if } l(Q_0) = l(Q_2) + 1 \geq 2. \end{cases}$$

Then, let $Q_3 = P(P_\phi, s_\phi, s_{r+1}^0)$, $Q_4 = P(P_\phi, s_\phi, s_{r+1}^1)$, $Q_5 = P(P_\phi, s_\phi, t_{r+1}^0)$, $Q_6 = P(P_\phi, s_\phi, t_{r+1}^1)$, $Q_7 = P(P_\phi, t_\phi, s_{r+1}^0)$, $Q_8 = P(P_\phi, t_\phi, s_{r+1}^1)$, $Q_9 = P(P_\phi, t_\phi, t_{r+1}^0)$, and $Q_{10} = P(P_\phi, t_\phi, t_{r+1}^1)$. Then, let

$$u = \begin{cases} s_\phi & \text{if } s_\phi \in \{s_{r+1}^0, s_{r+1}^1, t_{r+1}^0, t_{r+1}^1\}, \\ s_{r+1}^0 & \text{if } l(Q_3) < l(Q_4) \leq l(Q_5), \\ s_{r+1}^1 & \text{if } l(Q_4) < l(Q_3) \leq l(Q_5), \\ t_{r+1}^0 & \text{if } l(Q_5) < l(Q_6) \leq l(Q_3), \\ t_{r+1}^1 & \text{if } l(Q_6) < l(Q_5) \leq l(Q_3). \end{cases}$$

and

$$v = \begin{cases} t_\phi & \text{if } t_\phi \in \{s_{r+1}^0, s_{r+1}^1, t_{r+1}^0, t_{r+1}^1\}, \\ s_{r+1}^0 & \text{if } l(Q_7) < l(Q_8) \leq l(Q_9), \\ s_{r+1}^1 & \text{if } l(Q_8) < l(Q_7) \leq l(Q_9), \\ t_{r+1}^0 & \text{if } l(Q_9) < l(Q_{10}) \leq l(Q_7), \\ t_{r+1}^1 & \text{if } l(Q_{10}) < l(Q_9) \leq l(Q_7). \end{cases}$$

Let $u' = (u)^{\tau+1}$, $v' = (v)^{\tau+1}$, $x' = (x)^{\tau+1}$, $y' = (y)^{\tau+1}$, $G_2 = D_{\tau, n}^{(u')^1} \cup D_{\tau, n}^{(v')^1}$, and $G_3 = G_1/G_2$. By Lemma 6, there exists a Hamiltonian path P'_ϕ joining u' and v' in G_2 and exists a Hamiltonian path P'_{r+1} joining x' and y' in G_3 . We can confirm that $P_1, \dots, P_\phi = \langle P(P_\phi, s_\phi, u), P_\phi, P(P_\phi, v, t_\phi) \rangle, \dots$, and $P_{\tau+1} = \langle x, P_{r+1}, P(P_{\tau+1}, y, t_{r+1}) \rangle$ are $r+1$ disjoint paths such that $V(\bigcup_{i=1}^{r+1} P_i) = V(D_{\tau+1, n})$.

Case 3.1.2. $\phi \neq \varphi$. The case is similar to the Case 3.1.1, so we skip it.

Case 3.2. $q=1$. Choose x, y, s_ϕ , and t_ϕ such that $x \in V(G_0)$, $y \in V(G_1)$, $s_\phi, t_\phi \in W_3$, and $\phi \in \{1, 2, \dots, \tau+1\}$ with

$$x = \begin{cases} t_\phi & \text{if } s_\phi \in V(G_1), \\ s_\phi & \text{if } t_\phi \in V(G_1). \end{cases} \text{ and } y = \begin{cases} s_\phi & \text{if } s_\phi \in V(G_1), \\ t_\phi & \text{if } t_\phi \in V(G_1). \end{cases}$$

Let $S' = S \setminus \{s_\phi\}$ and $T' = T \setminus \{t_\phi\}$. According to the induction hypothesis, these exist r disjoint paths $P_1, P_2, \dots, P_i, \dots, P_{r+1}$ joining S' and T' such that $V(\bigcup_{i=1}^{r+1} P_i) = V(D_{\tau+1, n})$ with $i \neq \phi$. Choose P_φ with $x \in V(P_\varphi)$ and $\varphi \in \{1, 2, \dots, r+1\}$. Select two distinct nodes x^0 and x^1 such that (x^0, x) , $(x, x^1) \in E(P_\varphi)$. Let $Q_0 = P(P_\varphi, s_\varphi, x^0)$, $Q_1 = P(P_\varphi, s_\varphi, x^1)$, $Q_2 = P(P_\varphi, t_\varphi, x^0)$, and $Q_3 = P(P_\varphi, t_\varphi, x^1)$. Then, let

$$u = \begin{cases} s_\varphi & \text{if } s_\varphi \in \{x^0, x^1\}, \\ x^0 & \text{if } l(Q_0) < l(Q_1), \\ x^1 & \text{if } l(Q_1) < l(Q_0). \end{cases}$$

and

$$v = \begin{cases} t_\phi & \text{if } t_\phi \in \{x^0, x^1\}, \\ x^0 & \text{if } l(Q_2) < l(Q_3), \\ x^1 & \text{if } l(Q_3) < l(Q_2). \end{cases}$$

Let $u' = (u)^{\tau+1}$, $v' = (v)^{\tau+1}$, and $x' = (x)^{\tau+1}$. Then, we can claim the following cases with respect to u' , v' , x' , and y .

Case 3.2.1. $(u')_{\tau+1} \neq (v')_{\tau+1} \neq (x')_{\tau+1} \neq (y)_{\tau+1}$. Choose $\delta, \omega \in \{0, 1, \dots, N_\tau\}$ with $\delta, \omega \notin \{(u')_{\tau+1}, (v')_{\tau+1}, \lambda, (x')_{\tau+1}, (y)_{\tau+1}\}$.

Let $G_3 = D_{\tau,n}^{(x')_{\tau+1}} \cup D_{\tau,n}^{(y)_{\tau+1}} \cup D_{\tau,n}^\delta \cup D_{\tau,n}^\omega$ and $G_4 = G_0 \square (D_{\tau,n}^\lambda \cup G_3)$. By Lemma 6, there exist a disjoint path P'_ϕ covering all nodes in G_3 and a disjoint path $P'_{\tau+1}$ covering all nodes in G_4 . Moreover, let $P_{\tau+1} = \langle x, P'_{\tau+1} \rangle$ and $P_\phi = \langle P(P_\phi, s_\phi, u), P'_\phi, P(P_\phi, v, t_\phi) \rangle$. Then, P_1, P_2, \dots, P_{r+1} are $r+1$ disjoint paths in $D_{\tau+1,n}$, P_i joining s_i and t_i , $1 \leq i \leq \tau+1$, that cover all the nodes of $D_{\tau+1,n}$.

Case 3.2.2. $(u')_{\tau+1} = (y)_{\tau+1}$ or $(v')_{\tau+1} = (y)_{\tau+1}$ or $(x')_{\tau+1} = (y)_{\tau+1}$. The case is similar to the Case 3.2.1, so we skip it. $(u')_{\tau+1} = (y)_{\tau+1}$.

Case 3.3. $m' = 2$. The case is similar to the Case 3.2, so we skip it.

In summary, given a set of $r+1$ distinct sources $S = \{s_1, s_2, \dots, s_{r+1}\}$ and a set of $r+1$ distinct sinks $T = \{t_1, t_2, \dots, t_{r+1}\}$ in $D_{\tau+1,n}$ such that $S \cap T = \emptyset$, there always exist $r+1$ disjoint paths P_1, P_2, \dots, P_{r+1} in $D_{\tau+1,n}$,

joining s_i and t_i , $1 \leq i \leq \tau+1$, such that $V(\bigcup_{i=1}^{r+1} P_i) = V(D_{\tau+1,n})$.

Theorem 10. For $k \geq 0$ and $n \geq 2$, $D_{k,n}$ is a paired many-to-many r -disjoint path coverable with $1 \leq r \leq \lfloor (n+k-1)/2 \rfloor$ except for $D_{1,2}$.

Proof. By Lemma 4, the theorem holds with $k = 0$. By Lemma 8, the theorem holds with $D_{1,n}$. For $k > 1$ and $n \geq 2$, the theorem holds for $n+k$ is even by Lemma 9 and the theorem holds for $n+k$ is odd is similar to that of Lemma 9 and thus omitted.

Clearly, Theorem 10 implies the following theorem.

Theorem 11. For $k \geq 0$ and $n \geq 2$, $D_{k,n}$ is (unpaired) many-to-many r -disjoint path coverable with $1 \leq r \leq \lfloor (n+k-1)/2 \rfloor$ except for $D_{1,2}$.

Conclusions

DCell has been proposed for one of the most important data center networks and can support millions of servers with outstanding network capacity and provide good fault tolerance by only using commodity switches. In this paper, we give a construction scheme for many-to-many r -disjoint path covers in a k -dimensional DCell with n -port switches, $D_{k,n}$, with $1 \leq r \leq \lfloor (n+k-1)/2 \rfloor$.

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